

The Variation of Photon Speed with Photon Frequency in Quantum Gravity

Anuj Kumar Dubey¹, A. K. Sen² and Sonarekha Nath³

Department of Physics, Assam University, Silchar-788011, Assam, India

email: danuj67@gmail.com¹,
asokesen@yahoo.com², sonarekha92@gmail.com³

Date: 31 May 2016

Abstract

In the present work, we have derived an expression of Planck mass or Planck energy by equating the Compton wavelength with Kerr gravitational radius of the Kerr rotating body. Then we have derived the modified expression for the photon energy-momentum dispersion relation and hence derived the variation of the photon propagation speed with photon frequency. We have found that the photon propagation speed, depends on the frequency of the photon, polarization state of photon, the rotation parameter of the Kerr rotating body and also on the latitude. Quantum gravity effect could be seen from the derived results of the photon propagation speed.

Keywords: Quantum Gravity; Lorentz Invariance Violation; Photon Propagation Speed

1 Introduction

One of the most important problems of modern fundamental physics is the problem of reconciling classical general relativity, the theory of macroscopic gravitational phenomena, with quantum theory, so-called quantum gravity problem. Quantum gravity is requested to be the theory from which both quantum mechanics and general relativity should emerge in certain appropriate limits. Hence, it is natural to imagine that quantum gravity will offer a solution to the problems of quantum mechanics and general relativity [1].

There are many competing approaches of quantum gravity, such as string theory, loop quantum gravity, Euclidean path integrals, noncommutative geometry and others. Another recent theory of quantum gravity (E-gravity) proposes an anisotropy of space-time [2]. Till now do not have a complete consistent theory of quantum gravity.

As discussed in the book of Rovelli and Vidoto [3, 4], we can obtain the minimal size, where we can localize a quantum particle without having it hidden by its own horizon.

The minimal size (L), given as [3, 4]:

$$L = \frac{MG}{c^2} = \frac{EG}{c^4} = \frac{pG}{c^3} = \frac{\hbar G}{Lc^3} \quad (1)$$

Solving this for L , we find that it is not possible to localize anything with a precision better than the length which is called the Planck length (L_{Planck}) and given as:

$$L_{Planck} = \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-35} \text{ meter} \quad (2)$$

Similarly, we can make no smaller measurement of time than the Planck time (t_{Planck}) given as:

$$t_{Planck} = \frac{L_{Planck}}{c} = \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} \sim 5.4 \times 10^{-44} \text{ seconds} \quad (3)$$

The Planck mass (M_{Planck}) is given as:

$$M_{Planck} = \frac{\hbar}{c L_{Planck}} = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} \sim 2.2 \times 10^{-8} \text{ kilogram} \quad (4)$$

The Planck energy (E_{Planck}) is given as:

$$E_{Planck} = M_{Planck} c^2 = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}} \sim 1.2 \times 10^{28} \text{ ev} \quad (5)$$

The search for a quantum gravity theory has been one of the main aims of theoretical physics for many years by now. However the efforts in this direction have been often hampered by the lack of experimental and observational tests able to select among, or at least constrain, the numerous quantum gravity models proposed so far such as string theory and loop quantum gravity. This situation has changed in the last decade thanks to the realization that some quantum gravity inspired violations of Lorentz symmetry could be constrained using current experiments and observations [5].

A cornerstone of Einstein's special relativity is Lorentz invariance; the postulate that all observers measure exactly the same speed of light in vacuum, independent of photon-energy. While special relativity assumes that there is no fundamental length-scale associated with such invariance. There is a fundamental scale the Planck scale, ($L_{Planck} \approx 1.6 \times 10^{-35}$ meter or $E_{Planck} = M_{Planck} c^2 \approx 1.2 \times 10^{28}$ electron-volts), at which quantum effects are expected to strongly affect the nature of space-time. There is great interest in the idea that Lorentz invariance might break near the Planck scale. A key test of such violation of Lorentz invariance is a possible variation of photon speed with energy [6].

Some of the proposed models of quantum gravity predict a breakdown of the basic postulates in Einstein's theories (special and general relativity), such as the constancy of the speed of light in a vacuum. As a consequence of this breakdown, a major symmetry in special relativity the Lorentz symmetry that governs the momentum-energy relation in the microscopic world would also be violated [7].

Quantum gravity effect could be seen from dispersion relations violating Lorentz invariance. In order to explain these results, we can use dispersion relations coming from loop quantum gravity or other arguments based on effective field theories. Lorentz invariance is then seen as a good low-energy symmetry which may be violated at very high energies [8].

Although there is enormous uncertainty about the nature of quantum gravity, one thing is quite certain: the commonly used ideas of space-time should break down at or before the Planck length is reached. For example, elementary scattering processes with a Planck-sized center of mass energy create large enough quantum fluctuations in the gravitational field that space-time can no longer be treated as a classical continuum. It is then natural to question the exactness of the Lorentz invariance that is pervasive in all more macroscopic theories [9, 10].

Exact Lorentz invariance requires that an object can be arbitrarily boosted. Since the corresponding Lorentz contractions involve arbitrarily small distances, there is an obvious tension with the expected breakdown of classical space-time at the Planck length. Indeed, quite general arguments are made that lead to violations of Lorentz invariance within the two most popular approaches towards quantum gravity: string theory [11, 12] and loop quantum gravity [13, 14, 15]. This has given added impetus to the established line of research dedicated to the investigation of ways in which fundamental symmetries, like Lorentz invariance or CPT, could be broken [16, 17, 18, 19, 20].

It was realized that extremely precise tests could be made with a sensitivity appropriate to certain order of magnitude estimates of violations of Lorentz invariance [21]. The sensitivity of the tests arises because there is a universal maximum speed when Lorentz invariance holds, and even small modifications to the standard dispersion relation relating energy and 3-momentum give highly magnified observable effects on the propagation of ultra-relativistic particles.

With this background the present paper is organized as follows. In Section-2, we will discuss Lorentz invariance violation in quantum gravity and hence modified energy-momentum dispersion relation in case of photon. In Section-3, we will derive the expression for the variation of photon propagation speed with photon energy (or frequency). Finally, some conclusions are made in Section-4.

2 Lorentz invariance violation in quantum gravity and modified photon energy-momentum dispersion relation

The form of the standard energy-momentum dispersion relation is fixed by the Lorentz symmetry. Even though the Lorentz symmetry is one of the most important symmetries in nature, there are indications from various approaches to quantum gravity that the Lorentz symmetry might be violated in the ultraviolet limit. Thus, it is possible that the Lorentz symmetry is only an effective symmetry which holds in the infrared limit of quantum gravitational processes. As the standard energy-momentum dispersion relation depends on the Lorentz symmetry, it is expected that the standard energy-momentum dispersion relation will also get modified in the ultraviolet limit. In fact, it has been observed that such modification to the standard energy-momentum relation does occur in the discrete space-time, models based on string field theory, space-time foam, the spin-network in loop quantum gravity, non-commutative geometry, and Horava-Lifshitz gravity (as detailed in a paper [22] and cited references).

The modification of the standard energy-momentum dispersion relation has motivated the development of double special relativity [23]. In this theory, apart from the velocity of light being the maximum velocity attainable, there is also a maximum energy scale in nature. This energy scale is the Planck energy (E_{Planck}), and it is not possible for a particle to attain energies beyond this energy. The double special relativity has been generalized to curved space-time, and this doubly general theory of relativity is called gravity's rainbow [24]. In this theory, the geometry of space-time

depends on the energy of the test particle. So, particles of different energy see the geometry of space-time differently. Hence, the geometry of space-time is represented by a family of energy dependent metrics forming a rainbow of metrics. This is the reason the theory has been called gravity's rainbow. In order to construct this theory, the modified energy-momentum dispersion relation (MDR) in curved space-time is discussed in details in the works of Ali and Faizal [22, 25, 26, 27, 28, 29, 30] .

The modified energy-momentum dispersion relation is given as [22, 25, 26, 27, 28, 29, 30] :

$$E^2 f^2\left(\frac{E}{E_{Planck}}\right) - p^2 c^2 g^2\left(\frac{E}{E_{Planck}}\right) = m^2 c^4 \quad (6)$$

where E_{Planck} is the Planck energy.

The functions $f(\frac{E}{E_{Planck}})$ and $g(\frac{E}{E_{Planck}})$ are called the rainbow functions, and they are required to satisfy the following relations:

$$\lim_{\frac{E}{E_{Planck}} \rightarrow 0} f\left(\frac{E}{E_{Planck}}\right) = 1$$

$$\lim_{\frac{E}{E_{Planck}} \rightarrow 0} g\left(\frac{E}{E_{Planck}}\right) = 1$$

This condition is needed, as the theory is constrained to reproduce the standard dispersion relation in the infrared limit.

The simplest kinematic framework for Lorentz violation in particle based experiments is to propose modified dispersion relations for particles, while keeping the usual energy-momentum conservation laws. This was the approach taken in much of the work using astrophysical phenomena. In a given observers frame in flat space, this is done by postulating that the usual Lorentz invariant dispersion law $E^2 = m^2 c^4 + p^2 c^2$ is replaced by some function $E^2 = F(p, m)$. In general the preferred frame is taken to coincide with the rest frame of the cosmic microwave background. Since we live in an almost Lorentz invariant world (and are nearly at rest with respect to the CMBR), in the preferred frame $F(p, m)$ must reduce to the Lorentz invariant dispersion at small energies and momenta [31].

It is natural to expand $F(p, m)$ about $p = 0$, which yields the expression given as [31]:

$$E^2 = m^2 c^4 + p^2 c^2 + F_i^{(1)} p^i c + F_{ij}^{(2)} p^i p^j c^2 + F_{ijk}^{(3)} p^i p^j p^k c^3 + \dots \quad (7)$$

In the above equation (7) the constant coefficients $F_{ij\dots n}^{(n)}$ have dimension and arbitrary but presumably such that the modification is small. The order n of the first non-zero term in above equation (7) depends on the underlying model of quantum gravity taken [31].

Since the underlying motivation for Lorentz violation is quantum gravity, it is useful to factor out the Planck energy in the coefficients $F^{(n)}$ and rewrite above equation (7) given as [31]:

$$E^2 = m^2 c^4 + p^2 c^2 + E_{Pl} f_i^{(1)} p^i c + f_{ij}^{(2)} p^i p^j c^2 + \frac{f_{ijk}^{(3)}}{E_{Pl}} p^i p^j p^k c^3 + \dots \quad (8)$$

such that the coefficients $f^{(n)}$ are dimensionless.

In most of the literature a simplifying assumption is made that rotation invariance is preserved. In nature, we cannot have the rotation subgroup of the Lorentz group strongly broken while preserving boost invariance. Such a scenario leads immediately to broken rotation invariance at every energy which is unobserved. Hence, if there is strong rotation breaking there must also be a broken boost subgroup. However, it is possible to have a broken boost symmetry and unbroken rotation symmetry. Either way, the boost subgroup must be broken [31].

Phenomenologically, it therefore makes sense to look first at boost Lorentz violation and neglect any violation of rotational symmetry. If we make this assumption then the above equation (8) can be rewritten as:

$$E^2 = m^2 c^4 + p^2 c^2 + E_{Pl} f_i^{(1)} |p| c + f_{ij}^{(2)} p^2 c^2 + \frac{f_{ijk}^{(3)}}{E_{Pl}} |p|^3 c^3 + \dots \quad (9)$$

There is no a priori reason (from a phenomenological point of view) that the coefficients in equation (9) are universal (and in fact one would expect the coefficients to be renormalized differently even if the fundamental Lorentz violation is universal). We will therefore label each $f^{(n)}$ as $f_A^{(n)}$, where 'A' represents the particle species [31].

It is expected that the energy-momentum dispersion relation could be modified to include dependence on the ratio of the particle's energy (E) and the quantum gravity energy (E_{QG}) [32].

The space-time fuzziness effectively produces an uncertainty in the velocity of particles of order $\frac{E}{E_{QG}}$ [33].

In case of photon at small energies $E_{Ph} < E_{QG}$, we expect that a series expansion of the dispersion relation should be applicable given as [21, 17]:

$$c^2 p^2 = E_{Ph}^2 [1 + \xi \frac{E_{Ph}}{E_{QG}} + o(\frac{E_{Ph}}{E_{QG}})^2] \quad (10)$$

Here p is the photon momentum and c is the velocity of light. E_{Ph} is the photon energy and E_{QG} is an effective quantum gravity energy scale. The quantity $\xi (= \pm 1)$ is a sign ambiguity that would be fixed in a given dynamical framework.

Using above equation (10) and neglecting the second and higher order terms of $(\frac{E_{Ph}}{E_{QG}})$, we can write the expression of photon momentum (p) as:

$$p = \pm \frac{E_{Ph}}{c} \sqrt{1 + \xi \frac{E_{Ph}}{E_{QG}}} \quad (11)$$

According to the sign of $\xi (= \pm 1)$, the group velocity of high-energy photons could be sub or super-luminal, when defined in the usual way by $\frac{\partial E_{Ph}}{\partial p}$. Like with all quantum gravity effects, the suppression of Lorentz invariance violation by the ratio of the particle energy to the quantum gravity energy may appear discouraging at first sight [32].

3 The variation of photon propagation speed with photon frequency

Using equation (10) and neglecting the second and higher order terms of $(\frac{E_{Ph}}{E_{QG}})$, we can write the expression of the group velocity (energy-dependent velocities) of photons ($v_{ph} = \frac{\partial}{\partial p} E_{Ph}$) as:

$$v_{ph} = \frac{\partial}{\partial p} E_{Ph} \approx \frac{pc^2}{E_{Ph}(1 + \frac{3\xi E_{Ph}}{2E_{QG}})} \quad (12)$$

Using equation (11) in above equation (12), we can write:

$$v_{ph} \approx c \frac{\sqrt{(1 + \xi \frac{E_{Ph}}{E_{QG}})}}{(1 + \frac{3\xi E_{Ph}}{2E_{QG}})} \quad (13)$$

After simplification (using binomial approximation) and neglecting the second and higher order terms of $(\frac{E_{Ph}}{E_{QG}})$, the above equation (13) becomes:

$$v_{ph} \approx c(1 - \xi \frac{E_{Ph}}{E_{QG}}) \quad (14)$$

This type of velocity dispersion results from a picture of the vacuum as a ‘quantum-gravitational medium’, which responds differently to the propagation of particles of different energies and hence velocities [21].

Some quantum-gravity theories [6, 34, 35] are consistent with the photon-propagation speed (v_{ph}) varying with photon energy (E_{ph}), and becoming considerably different from the low-energy limit of speed of light, $c \equiv v_{ph} (E_{ph} \rightarrow 0)$, near the Planck scale (when E_{ph} becomes comparable to $E_{Planck} (= M_{Planck} c^2)$) [6].

The above equation (14) encodes a minute modification for most practical purposes, as quantum gravity energy (E_{QG}) is believed to be a very high scale, presumably of the order of the Planck scale. Even so, such a deformation could be rather significant for even moderate-energy signals, if they travel over very long distances. According to equation (15), a signal of energy (E) that travels a distance (L) acquires a time delay, measured with respect to the ordinary case of an energy independent speed c for massless particles:

$$\Delta t = \xi \frac{\Delta E}{E_{QG}} (\frac{L}{c}) \quad (15)$$

If the tiny effect on the speed of light accumulates as high energy photons travel cosmic distances, the spectra of γ -ray bursts (GRB) would reveal an energy-dependent speed of light through a measurable difference of the time of arrival of high and low energy photons. Amelino-Camelia et al. [21] noted that photons ($m = 0$) with different energies would travel with different velocities. For a γ -ray bursts originating at a distance (L) from us, the difference in time of arrival of different energy components would be Δt ($\approx \xi \frac{\Delta E}{E_{QG}} (\frac{L}{c})$). If the parameter ξ were of order 1 and $L \sim 100$ Mpc, then for $\Delta E \sim 100$ MeV, we would have $\Delta t \sim 10^{-2}s$, making it close to measurable in γ -ray bursts.

Using high-energy observations from the Fermi Large Area Telescope (LAT) of γ -ray burst GRB090510, a model has been tested in which photon speeds are distributed normally around c with a standard deviation proportional to the photon energy. The model's characteristic energy scale is constrained beyond the Planck scale at $> 2.8 E_{Planck}$ ($> 1.6 E_{Planck}$), at 95 % (99 %) confidence [36].

As detailed in our previous works Dubey and Sen 2015 [39, 40], covariant form of metric tensor for Kerr family (Kerr 1963 [37], Newman et al. 1965 [38]) in terms of Boyer-Lindquist coordinates with signature (+,-,-,-) is expressed as [39, 40, 41]:

$$ds^2 = g_{tt}c^2dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 + 2g_{t\phi}cdtd\phi \quad (16)$$

where g_{ij} 's are non-zero components of Kerr family.

Non-zero components g_{tt} of Kerr metric is given as follows [39]:

$$(g_{tt})_{Kerr} = \left(1 - \frac{r_g r}{r^2 + a^2 \cos^2 \theta}\right) \quad (17)$$

Here $r_g (= 2GM/c^2)$ and $a (= \frac{J}{Mc})$ are Schwarzschild radius and rotation parameter respectively.

The above equation (17) can be written as:

$$(g_{tt})_{Kerr} = 1 - \frac{r_g r}{r^2(1 + \frac{a^2}{r^2} \cos^2 \theta)} = 1 - \frac{r_g}{r} \left(1 + \frac{a^2}{r^2} \cos^2 \theta\right)^{-1} \quad (18)$$

If we consider $a \ll r$, and using binomial approximation and keeping only the term of order of $\frac{a^2}{r^2}$, then the above equation (18) becomes:

$$(g_{tt})_{Kerr} = 1 - \frac{r_g}{r} \left(1 - \frac{a^2}{r^2} \cos^2 \theta\right) \quad (19)$$

If we put rotation parameter (a) equals to zero in the above equation (19), then the component g_{tt} of Schwarzschild metric is given as:

$$(g_{tt})_{Schwarzschild} = 1 - \frac{r_g}{r} \quad (20)$$

Now if we compare the Non-zero component g_{tt} of Kerr metric (as given by equation (19)) with the component g_{tt} of Schwarzschild metric (as given by equation (20)), then we can conclude that

$$(r_g)_{Kerr} = r_g \left(1 - \frac{a^2}{r^2} \cos^2 \theta\right) \quad (21)$$

Here the term $(r_g)_{Kerr}$ is said to be Kerr gravitational radius for rotating body, analogous to the Schwarzschild gravitational radius $r_g (= 2GM/c^2)$ for static body. As we know the difference between Kerr metric and Schwarzschild metric is only lies due to the effect of rotation. If the rotation parameter (a) goes to zero then from above equation (21), we can conclude that

$$(r_g)_{Kerr \text{ or Rotation}} = (r_g)_{Schwarzshild \text{ or Static}}, \text{ if } a = 0 \quad (22)$$

In other words we can say that

$$(r_g)_{Kerr \text{ or } Rotation} = (r_g)_{Schwarzshild \text{ or } Static} \left(1 - \frac{a^2}{r^2} \cos^2 \theta\right) \quad (23)$$

The above equation (23) clearly indicates that the effect of rotation is to reduce the Schwarzschild gravitational radius of the static black hole.

Most of the authors have calculated the Planck mass (M_{Planck}) by considering a static body whose Compton wavelength equals to its Schwarzschild (or gravitational) radius [43, 44, 45]. Then this equivalence gives us [43];

$$\frac{\hbar}{Mc} = \frac{2GM}{c^2} = r_g$$

from which it follows that

$$(M_{Planck})_{Schwarzshild \text{ or } Static} = \sqrt{\frac{\hbar c}{2G}}$$

Now, if we consider a rotating body whose Compton wavelength equals to its Kerr gravitational radius. Then this equivalence gives us;

$$\frac{\hbar}{Mc} = \frac{2GM}{c^2} \left(1 - \frac{a^2}{r^2} \cos^2 \theta\right) = (r_g)_{Kerr \text{ or } Rotation} \quad (24)$$

from which it follows that

$$(M_{Planck})_{Kerr \text{ or } Rotation} = \sqrt{\frac{\hbar c}{2G}} \left(1 - \frac{a^2}{r^2} \cos^2 \theta\right)^{-1/2} \quad (25)$$

If we consider $a \ll r$, and using binomial approximation, keeping only the term of order of $\frac{a^2}{r^2}$ and neglecting the higher order terms of $\frac{a^2}{r^2}$, then above equation (25) becomes:

$$(M_{Planck})_{Kerr \text{ or } Rotation} = \sqrt{\frac{\hbar c}{2G}} \left(1 + \frac{a^2}{2r^2} \cos^2 \theta\right) \quad (26)$$

Now the Planck mass $(M_{Planck})_{Kerr \text{ or } Rotation}$ as given by the above equation (26) is obtained by considering a rotating body whose Compton wavelength equals to its Kerr gravitational radius.

Now we can write the quantum gravity energy $(E_{QG})_{Kerr \text{ or } Rotation}$ as:

$$(E_{QG})_{Kerr \text{ or } Rotation} = (M_{Planck})_{Kerr \text{ or } Rotation} c^2 = \sqrt{\frac{\hbar c^5}{2G}} \left(1 + \frac{a^2}{2r^2} \cos^2 \theta\right) \quad (27)$$

Now using above equation (27), momentum of photon equation (11) can be rewritten as:

$$p = \pm \frac{E_{Ph}}{c} \sqrt{\left(1 + \xi \frac{E_{Ph}}{\sqrt{\frac{\hbar c^5}{2G}} \left(1 + \frac{a^2}{2r^2} \cos^2 \theta\right)}\right)} \quad (28)$$

Here $E_{ph}(= \hbar\omega)$ is the energy. \hbar and ω are Plancks constant and angular frequency of photon respectively. Using equation (28), group velocity of photon (v_{ph}) given by equation (14), can be written as:

$$v_{ph} = \frac{\partial}{\partial p} E_{Ph} \approx c(1 - \xi \frac{\hbar\omega}{\sqrt{\frac{\hbar c^5}{2G}} (1 + \frac{a^2}{2r^2} \cos^2\theta)}) \quad (29)$$

After simplification the above equation (29) becomes:

$$v_{ph} \approx c(1 - \xi \frac{\omega}{c} \sqrt{\frac{2\hbar G}{c^3}} (\frac{1}{1 + \frac{a^2}{2r^2} \cos^2\theta})) = c(1 - \xi \frac{\omega}{c} \frac{L_{Planck}}{1 + \frac{a^2}{2r^2} \cos^2\theta}) \quad (30)$$

From the above equation (30) of photon propagation speed (v_{ph}), we can see that the photon propagation speed depends on the frequency of the photon, the rotation parameter of the Kerr rotating body and also on the latitude (θ).

The presence of ' θ ' can be explained in the following way: when light is circularly polarized $\cos^2\theta = 1$ and when light is unpolarized $\cos^2\theta = 0$. Thus we obtain from equation (30), two different dispersion relation for circularly polarized light and unpolarized light. This is a new property of lorentz invariance, which is not reported by any researcher in past.

If we consider the rotation parameter (a) equals to zero, or the latitude ($\theta = \frac{\pi}{2}$, at equator), then the above equation (30) can be rewritten as:

$$v_{ph} \approx c(1 - \xi \frac{\omega}{c} L_{Planck}) \quad (31)$$

If we consider the latitude, ($\theta = 0$) for North pole, and ($\theta = \pi$) for South pole, then the above equation (30) can be rewritten as:

$$v_{ph} \approx c(1 - \xi \frac{\omega}{c} \frac{L_{Planck}}{1 + \frac{a^2}{2r^2}}) \quad (32)$$

The presence of the Planck length ($L_{Planck} = \sqrt{\frac{2\hbar G}{c^3}}$) in the expressions (30), (31) and (32) of the photon propagation speed (v_{ph}), clearly indicates us the effect of quantum gravity.

In general the rotation parameter of the Planck sized Kerr rotating body should be quantized as suggested in the references [41, 47, 48]. Notice that, in the present work, we have used the semi-classical theory, for deriving the expression (30) of the photon propagation speed. We have also considered the classical rotation parameter. To study completely the variation of photon propagation speed, we wish to use the quantized rotation parameter of planck sized Kerr rotating body in our future publication.

4 Conclusions

We can conclude from the present work that,

- We have derived the expression of Planck mass or Planck energy by considering the equivalence of Compton wavelength with Kerr gravitational radius of the Kerr rotating body. If we consider the rotation parameter equals to zero, then our derived results for Planck mass or Planck energy is matched with the earlier works in the literature (based on the consideration of the equivalence of Compton wavelength with Schwarzschild gravitational radius of the static body).
- We have derived the modified expression for the photon energy-momentum dispersion relation and hence derived the expression of the variation of the photon propagation speed with photon frequency.
- We have found that the photon propagation speed, depends on the frequency of the photon, the rotation parameter of the Kerr rotating body and also on the latitude.
- In addition to the above effect, we find that the dispersion relation or the Lorentz invariance is affected by the polarization state of the photon - an effect which was not previously reported by any author.
- The presence of the Planck length in the derived expression of the variation of the photon propagation speed, clearly indicates us the quantum gravity effect.

References

- [1] Ashtekar A., Introduction to loop quantum gravity: Quantum Gravity and Quantum Cosmology, Vol. 9999999 of the series Lecture Notes in Physics, pp 31-56, Springer Berlin Heidelberg (2013). arXiv:1201.4598v1 [gr-qc]
- [2] Linker P., E-gravity theory, The Winnower, 3:e145441.18359 (2016). DOI:10.15200/winn.145441.18359
- [3] Rovelli C. and Vidotto F., Covariant loop quantum gravity, An elementary introduction to quantum gravity and spin foam theory, Cambridge University Press, (2015).
- [4] Vidotto F., Atomism and relationalism as guiding principles for quantum gravity, PoS (FFP14) 222, (2014). arXiv:1309.1403v1 [physics.hist-ph]
- [5] Liberati S., Quantum gravity phenomenology via Lorentz violations, PoS (P2GC) 018, (2007). arXiv:0706.0142v1 [gr-qc]
- [6] Abdo A. A. et al., A limit on the variation of the speed of light arising from quantum gravity effects, Nature, **462**, 331-334 (2009).
- [7] Jacholkowska A., Spacetime fuzziness in focus, Nature Physics, **11**, 302-303, (2015).
- [8] Carmona J. M., Noncommutativity in field space and Lorentz invariance violation, Physics Letters B, **565**, 222-228, (2003).

- [9] Collins J., Perez A., Sudarsky D., Lorentz Invariance Violation and its Role in Quantum Gravity Phenomenology, arXiv:hep-th/0603002v1
- [10] Oriti D., Approaches to quantum gravity, Toward a new understanding of Space, Time and Matter, Cambridge University Press, (2009).
- [11] Ellis J. R., Mavromatos N. E. and Nanopoulos D. V., Quantum-gravitational diffusion and stochastic fluctuations in the velocity of light, Gen. Rel. Grav., **32**, 127, (2000).
- [12] Ellis J. R., Mavromatos N. E. and Nanopoulos D. V., A microscopic recoil model for light-cone fluctuations in quantum gravity, Phys. Rev. D, **61**, 027503, (2000).
- [13] Gambini R. and Pullin J., Nonstandard optics from quantum spacetime, Phys. Rev. D, **59**, 124021, (1999).
- [14] Alfaro J., Morales-Tecotl H. A. and Urrutia L. F., Quantum gravity corrections to neutrino propagation, Phys. Rev. Lett., **84**, 2318, (2000).
- [15] Morales-Tecotl H. A. and Urrutia L. F., Loop quantum gravity and light propagation, Phys. Rev. D., **65**, 103509, (2002).
- [16] Kosteletsky V. A. and Samuel S., Gravitational phenomenology in higher-dimensional theories and strings, Phys. Rev. D, **40**, 1886, (1989).
- [17] Tawfik A. N. and Diab A. M., Review on generalized uncertainty principle, Rep. Prog. Phys., **78**, 126001, (2015).
- [18] Kosteletsky V. A. and Potting R., CPT and Strings, Nucl. Phys. B, **359**, 545, (1991).
- [19] Kosteletsky V. A. and Potting R., CPT, strings, and meson factories, Phys. Rev. D, **51**, 3923, (1995).
- [20] Kosteletsky V. A. and Potting R., Expectation values, Lorentz invariance and CPT in the open bosonic string, Phys. Lett. B, **381**, 89, (1996).
- [21] Amelino-Camelia Giovanni, Ellis J., Mavromatos N. E., Nanopoulos D. V. and Sarkar S., Tests of quantum gravity from observations of γ -ray bursts, Nature, **393**, 763-765 (1998).
- [22] Hendi S. H. and Faizal M., Black holes in Gauss-Bonnet gravity's rainbow, Phys. Rev. D, **92**, 044027 (2015). arXiv:1506.08062v2 [gr-qc]
- [23] Magueijo J., and Smolin L., String theories with deformed energy-momentum relations, and a possible nontachyonic bosonic string, Phys. Rev. D, **71**, 026010, (2005).
- [24] Magueijo J., and Smolin L., Gravity's rainbow, Class. Quant. Grav., **21**, 1725, (2004).

- [25] Ali A. F., Faizal M. and Khalil M. M., Remnants of black rings from gravity's rainbow, *JHEP*, **1412**, 159, (2014).
- [26] Ali A. F., Faizal M. and Khalil M. M., Remnants for all black objects due to gravity's rainbow, *Nucl. Phys. B*, **894**, 341, (2015).
- [27] Ali A. F., Faizal M. and Khalil M. M., Absence of black holes at LHC due to gravity's rainbow, *Phys. Lett. B*, **743**, 295, (2015).
- [28] Ashour A., Faizal M., Ali A. F. and Hammad F., Branes in Gravity's Rainbow, *Eur. Phys. J.*, **C76**, 264, (2016). arXiv:1602.04926v2 [hep-th]
- [29] Ali A. F., Faizal M., Majumder B. and Mistry R., Gravitational collapse in gravity's rainbow, *Int. J. Geom. Meth. Mod. Phys.*, **12**, 1550085, (2015).
- [30] Ali A. F., Faizal M. and Majumder B., Absence of an effective Horizon for black holes in Gravity's Rainbow, *Europhys. Lett.* **109**, 20001, (2015).
- [31] Mattingly D., Modern tests of Lorentz invariance, *Living Rev. Relativity*, **8**, 5, (2005). arXiv:gr-qc/0502097v2
- [32] Girelli F., Hinterleitner F. and Major S. A., Loop quantum gravity phenomenology: Linking loops to observational physics, *SIGMA*, **8**, 098, (2012). arXiv:1210.1485v2 [gr-qc]
- [33] Amelino-Camelia Giovanni, Quantum-Spacetime Phenomenology, *Living Rev. Relativity*, **16**, 5, (2013).
- [34] Kostelecky V. A. and Mewes M., Astrophysical tests of Lorentz and CPT violation with photons, *Astrophys. J.*, **689**, L1-L4, (2008).
- [35] Amelino-Camelia G. and Smolin L., Prospects for constraining quantum gravity dispersion with near term observations, *Phys. Rev. D*, **80**, 084017, (2009).
- [36] Vasileiou V. et al., A Planck-scale limit on spacetime fuzziness and stochastic Lorentz invariance violation, *Nature Physics*, **11**, 344-346, (2015).
- [37] Kerr R. P., Gravitational field of a spinning mass as an example of algebraically special metrics, *Phys. Rev. Lett.*, **11**, 5, 237-238, (1963).
- [38] Newman E. T., Couch E., Chinnapared K., Exton A., Prakash A. and Torrence R., Metric of a rotating, charged mass, *Journal of Mathematical Physics*, **6**, 918-919, (1965).
- [39] Dubey A. K. and Sen A. K., An analysis of gravitational redshift from rotating body, *International Journal of Theoretical Physics*, **54**, 2398-2417, (2015).
- [40] Dubey A. K. and Sen A. K., Gravitational redshift in Kerr-Newman geometry, *Astrophysics and Space Science*, **360**, 29, (2015).
- [41] Bekenstein J. D. , Black holes and entropy, *Phys. Rev. D*, **7**, 8, (1973).

- [42] Carroll S. M., Spacetime and geometry: An introduction to general relativity, Vol. 1, Pearson, Addison Wesley, (2004).
- [43] Ha Y. K., Are black holes elementary particles ? Int. J. Mod. Phys. A, **24**, 18-19, 3577-3583, (2009).
- [44] Hoyle F., Burbidge G. and Narlikar J., A different approach to cosmology, Cambridge University Press, (2000).
- [45] Crothers Stephen J. and Dunning-Davies J., Planck particles and quantum gravity, Progress in Physics, **3**, (2006).
- [46] Hooft G. 't, On the quantum structure of a black hole, Nucl. Phys. B, **256**, 727 (1985).
- [47] Hod S., Bohr's correspondence principle and the area spectrum of quantum black holes, Phys. Rev. Lett., **81**, 20, 4293, (1998).
- [48] Ropotenko K., Quantization of the black hole area as quantization of the angular momentum component, Phys. Rev. D, **80**, 044022, (2009).